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NRL Report 5626

**A NOTE ON SELECTION OF AN ATMOSPHERIC
REFRACTIVITY MODEL FOR
RADAR RANGE-HEIGHT-ANGLE CHARTS**

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ABSTRACT

Work during the past few years by Bauer et al. of MIT Lincoln Laboratory and Bean and Thayer of the NBS Central Radio Propagation Laboratory (CRPL) has established the superiority of a negative-exponential model of the atmospheric radio refractivity vs height function, compared to the linearly decreasing refractivity assumed by the well-known $4/3$ -earth's-radius method of accounting for ray bending. However various values of the zero-altitude refractivity and the exponential constant may be used in the exponential model. For many purposes, such as plotting radar coverage on a range-height-angle chart, a standard assumption for the atmospheric refraction, corresponding to fixed values of these constants, is desirable. Various factors relevant to selection of such a standard are discussed, and it is concluded that the CRPL Exponential Reference Atmosphere, for a surface value of refractivity $N_0 = 313$, is a suitable model. A chart and table of ray-path range-height values for this model are given.

PROBLEM STATUS

The work described in this report is part of a more comprehensive and continuing project. This is a final report on this phase of the project.

AUTHORIZATION

NRL Problem R02-05
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and
NRL Problem R02-06
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A NOTE ON SELECTION OF AN ATMOSPHERIC REFRACTIVITY
MODEL FOR RADAR RANGE-HEIGHT-ANGLE CHARTS

INTRODUCTION

Until about three years ago, the general practice for calculating the radar range-height-angle relationship under standard atmospheric conditions was to follow the method of Schelleng, Burrows, and Ferrel, which they proposed in 1933. This method, known as the 4/3-earth's-radius principle, is described in standard texts on radio engineering. The basic assumption of the method is that the atmospheric refractive index decreases linearly with height. As has been recognized for some time by many workers, this assumption leads to serious errors at long ranges and low elevation angles. To avoid these errors, Bauer et al. (1) proposed in 1958 a negative-exponential model of the refractivity-height function. They made calculations of ray paths as a function of range and height, with initial ray angle as a parameter, for the following specific model of the refractive index:

$$n(h) = 1 + 0.000320 \exp (-0.03709 h) \quad (1)$$

where h is height in thousands of feet. This expression will hereinafter be referred to as Bauer's model.

The constants of this model were chosen to approximate atmospheric conditions in the region of Washington, D.C., in April. Bauer also gave constants for similar models applicable to January and July conditions at the same location.

Ground-to-air, air-to-ground, or ground-to-ground propagation was assumed, and the same assumption is implicit throughout the present report. That is to say, one terminal of the path is assumed to be not more than a few hundred feet above the earth's surface. (Possibly 1000 feet would be a suitable arbitrary limit.) Here and in all of the discussion that follows, the initial ray angle is the angle made by the radio ray with the tangent to the earth's (idealized) surface when the ray height is zero. The "range" from this zero point of the ray to its position at a specified height is the distance measured along the actual ray path. Thus these two quantities correspond to the angle and range of an elevated radar target as observed by a radar located on the earth's surface. If a radar antenna is located at an appreciable height above the earth's surface, but nevertheless low enough to qualify as ground-based, this height should in principle be added to the computed ray heights. (Bauer's calculations assumed an antenna height of 168 feet, so correction to his figures should be made for the difference between the actual antenna height and this assumed height. In the other calculations of ray paths considered in this report, the ray height is expressed relative to the antenna, or origin of the ray.)

The ray paths computed for measured atmospheric conditions were compared by Bauer with the purely theoretical results, and it was shown that the agreement was very good, although for Bauer's April model the disagreement was significant in warm humid weather at angles below 2 degrees, as might be expected. Obviously no single model will fit all possible atmospheric conditions, although above 2 degrees the April model works very well at all seasons. Overall, the exponential model was shown to be greatly superior to the 4/3-earth's-radius assumption.

To facilitate plotting radar coverage diagrams, and for other similar applications, a range-height chart is desired in which the ray paths appear as straight lines, although in the actual atmosphere they curve downward. Such a plot is automatically obtained with the Schelleng-Burrows-Ferrell method if the earth's surface is plotted with a radius of curvature equal to $4/3$ the true value, hence the name $4/3$ -earth's-radius principle. A chart having this straight-ray-path property is not as simply obtained for the exponential refractivity model, but one has been devised and described by Clarke and Blake (2), and applied to the results calculated by Bauer. The resulting chart has been incorporated into a method of range and coverage calculation (3). The range, height, and angle limits of that chart were 300 nautical miles, 100,000 feet, and 30 degrees. This height limit represents the maximum of the values published by Bauer.

A comprehensive study of atmospheric refractivity models has been made by Bean and Thayer (4), who have presented the results of extensive calculations of ray paths (5) for exponential models with different values of the surface refractivity and exponential constant. For general radio and radar engineering purposes, however, it is desirable to have a single specific model, to be used without regard to season or geographic location. The selection of such a model is the problem considered in this report.

Some of the views expressed as to criteria for selecting a model are controversial, and the reader is cautioned that ultimately a standard model or models of atmospheric refraction other than the one suggested here may be adopted by the engineering profession. In the meantime, however, there is a great need for an immediate interim standard, in the author's opinion. The former standard, based on the $4/3$ -earth's-radius principle, has been shown to be unacceptable for long-range low-elevation-angle applications. The general form of an improved model has been established, but a specific model has not been adopted as a standard. This report proposes a model to meet this interim need.

SELECTION OF A SPECIFIC EXPONENTIAL MODEL

The author has been advised by B. R. Bean* of the National Bureau of Standards Central Radio Propagation Laboratory, Boulder, Colorado, that the model

$$n(h) = 1 + 0.000313 \exp(-0.04385 h) \quad (2)$$

where again h is in thousands of feet, is based on a surface value of refractivity† obtained by averaging about 2×10^6 observations from about 70 weather stations over the United States for a period of 8 years. The exponential constant, 0.04385, originally given by Bean as 0.143859 for h in kilometers, conforms to the pattern described (4,5) as the CRPL Exponential Reference Atmosphere, which has been designed to agree with observed values. This model, Eq. (2), will hereinafter be referred to as the CRPL model.

The average altitude of the weather stations whose observations were averaged is about 700 feet. For naval use, a model based on sea-level conditions would seem more appropriate. Also, a model that is some sort of an average over the whole world, rather than the United States, would be desirable. However, choosing a single model acceptable to the entire radio engineering community is of even greater importance than choosing one especially suited to naval shipboard applications, to provide a common basis for specifying such things as the vertical coverage pattern of a radar. If such a common basis is established, misunderstandings are avoided, and even though this basis does not apply exactly to all parts of the world at all times, coverage plotted on the standard basis can be correctly interpreted, and revised to apply to special conditions if necessary. Of course, it is nevertheless desirable that the standard be as representative as possible of typical or average conditions.

*In a private communication dated December 8, 1960.

†Refractivity is here defined, as elsewhere, as $N(h) = [n(h) - 1] \cdot 10^6$. Hence the surface refractivity, $N_s = N(0)$, in the model of Eq. (2) is 313.

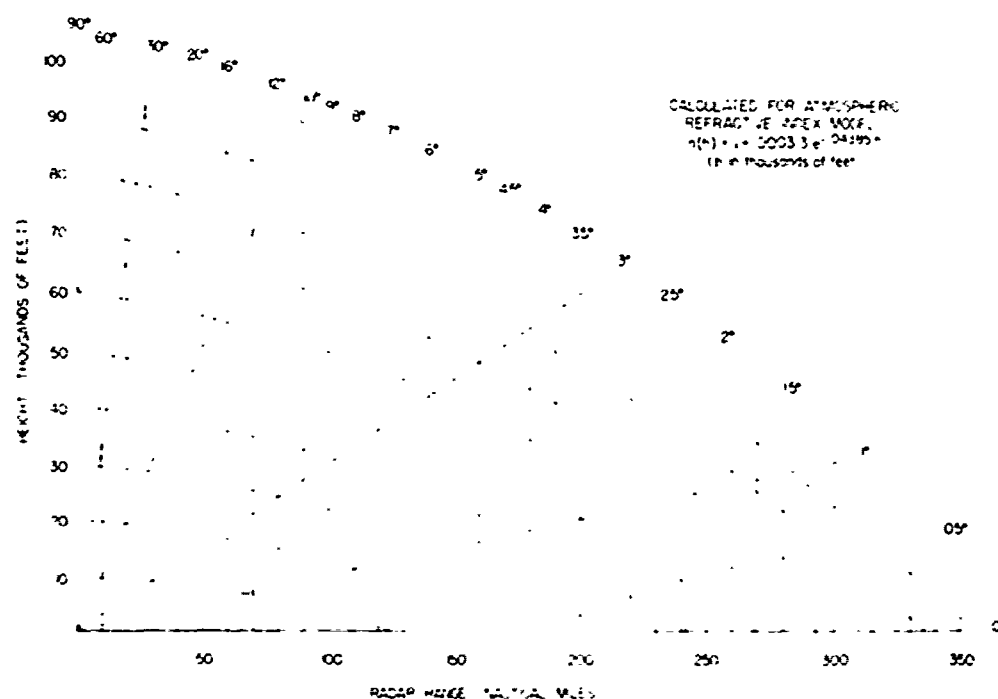


Fig. 1 - Straight-ray-path plot of range-height-angle values for the CRPL Exponential Reference Atmosphere, $N_2 = 513$

On the basis of this type of reasoning, this CRPL model was used for a radar range-height-angle straight-ray-path chart included as part of a recent paper on radar range calculation (6). This chart is also shown in the present report, as Fig. 1.

Bean and Thayer (5), in NBS Monograph 4, have published tables of values for this model. The units used are kilometers and milliradians. For most military applications a chart is desired with range in nautical miles, height in thousands of feet, and angle in degrees. To obtain the required values for plotting Fig. 1 from the NBS tables would have required interpolation, with possible loss of accuracy. Since the digital computer program for ray-path calculation had already been set up before existence of the NBS tables became known, the values for Fig. 1 were obtained by direct calculation with the NRL NAREC computer. The basic theory of such calculations is described by Bauer (1), Bean and Thayer (4), and others.

Although Fig. 1 extends only to an altitude of 100,000 feet and to 350 miles, values were calculated up to an altitude of 10⁶ feet, corresponding to a range of 1120 nautical miles at zero-degree elevation angle. Table 1 contains the entire set of calculated values. It should be noted that the altitude of 10⁶ feet, which is about 165 nautical miles, is well above the lower limit of the ionosphere, so that the ray paths computed are not correct above this limit at frequencies affected by the ionosphere. That is, values in Table 1 for altitudes above about 250,000 feet are not correct, or, at the lower frequencies. Above about 1000 megacycles the ionosphere has no appreciable effect, ordinarily. However, Table 1 can probably be used in its entirety with negligible error due to the ionosphere above about 500 megacycles (7).

It is intended that a chart will be constructed, in the near future, that will make use of this full set of values, preserving the straight-ray-path feature, with partially logarithmic range and height scales. Such a chart should have acceptable accuracy at all range and height values without excessive overall physical size.

During preparation of this report the author has learned that Dean Moore* of Gillilan Bros., Inc., Los Angeles, Calif., has independently selected the same atmospheric refractivity model as a standard for use in a radar development project. Moore has also constructed a straight-line-ray-path chart patterned after the one based on Bauer's model (2), for this CRPL model, extending to 450 nautical miles and an altitude of 200,000 feet. A copy of another chart based on Bauer's results, extended to a range of 450 miles and an altitude of 150,000 feet, has been received from A. J. Orlando† of Lockheed Electronics, Plainfield, N. J. The additional values are computed in accordance with Appendix A of Bauer's report (1).

COMPARISON WITH OTHER MODELS

Comparison of ray path values based on the two models (Bauer's and CRPL) indicates that the difference are not great. For example at 350-mile range the zero-degree ray has an altitude of about 90,000 feet for Bauer's model and about 87,000 feet for the CRPL model. The ray height computed at this range for the 4/3-earth's-radius model is approximately 81,000 feet. Thus the CRPL model produces slightly greater low-angle ray bending than Bauer's model in spite of its slightly smaller surface refractivity, because of its greater exponential constant. It is therefore a somewhat better compromise between cool or cold weather and warm weather conditions than is Bauer's April model.

Bauer's model conforms quite closely to the 4/3-earth's-radius model at low altitudes. This conformance is regarded in some circles as a virtue in an exponential model for general use, because it provides for an overlap between the new and old models and allows use of either model at low altitudes with the same results. In fact, according to L. R. Bean,‡ the International Radio Consultative Committee of the International Telecommunication Union has recommended a model based on this criterion. This model is also a CRPL Exponential Reference Atmosphere with different values of the constants than those of Eq. (2); when expressed for h in thousands of feet it is

$$n(h) = 1 + 0.000289 \exp (-0.04145 h). \quad (3)$$

It is thus evident that conformance to the 4/3-earth's-radius principle at low altitude may be achieved in an exponential model with many different combinations of the surface refractivity and exponential constant, Eq. (3) having the particular values that also conform to the pattern of the CRPL Exponential Reference Atmosphere.

The asserted advantage of this conformance is that the 4/3-earth's-radius principle could still be used in ground-wave (low altitude) calculations and the results would be the same as if the exponential model were used. But, the results would disagree only slightly with those of the $N_g = 313$ CRPL model, and if this disagreement is deemed serious, it could be eliminated by changing the 4/3 factor to a value that would produce agreement. The necessary factor would be about 1.4, corresponding to a 7/5-earth's-radius principle. In the author's opinion, the primary consideration should be the statistics of the actual atmosphere. As shown by Fig. 9 of Bean and Thayer's paper (4), the 4/3-earth's-radius principle "is systematically in disagreement with average bending; at low heights it gives too little bending, and at high altitudes it gives too much." It is possibly a good representation of average bending in the first few thousand feet of atmosphere, but the 7/5-earth's-radius would probably be better for very low-altitude calculations.

*Private communication dated March 3, 1961.

†Private communication dated March 23, 1961.

‡Private communication dated December 8, 1960.

Thus ruling out conformance to the 4/3-earth's-radius principle at zero height as an absolute criterion, the CRPL model, Eq. (2), seems to be a good choice for general radar coverage calculations. It is representative of average conditions over the United States, and since the United States is intermediate in location between tropic and arctic regions, it is probably a fair average for all latitudes.

There are, however, some objections to the CRPL model of Eq. (2) other than its nonconformance to the 4/3-earth's-radius principle. As pointed out to the author by J. R. Bauer² of MIT Lincoln Laboratory, the CRPL model of Eq. (2) is based on an average which includes summer conditions, often characterized by a highly erratic layer structure of the first few thousand feet above the surface, and by abnormally high n -gradients. Under these conditions, the standard deviation of ray-path height and the total ray bending are both considerably greater than in cooler weather, as Bauer has shown (1). Hence a model based on this kind of an average has less value for accurate target-height determination at any time than one based on more restricted conditions. Bauer feels that inclusion of summertime data in the averaging results in an unrealistically high value of the exponential constant. To avoid the degradation of predictability of a cooler-weather model that results when a year-round average is taken, he suggests the possibility of having separate models for warm and cool seasons. A cool-weather model which gives good prediction accuracy over a large part of the year in temperate climates, and also in warm weather at angles above about two degrees, is shown in Fig. 2. This chart represents Bauer's model, Eq. (1), and was constructed on the basis of his published ray-path calculations (1,2).

Bauer's arguments are impressive, and they emphasize the controversial nature of this matter as mentioned in the introduction. In evaluating some of these arguments, it should be realized that the application contemplated for charts of the type of Figs. 1 and 2 is the plotting of radar coverage diagrams, without specific reference to the time and place. Such a chart is not intended for radar height finding at a particular time and place, and should not be so used. That application requires a number of models from which one may be chosen to suit the specific conditions observed or estimated to exist. Statistical averages over all times and places are inappropriate for the purpose. But as a basis for a standard radar coverage plotting chart, a statistical-average model is possibly more appropriate than one that applies to a particular time and place.

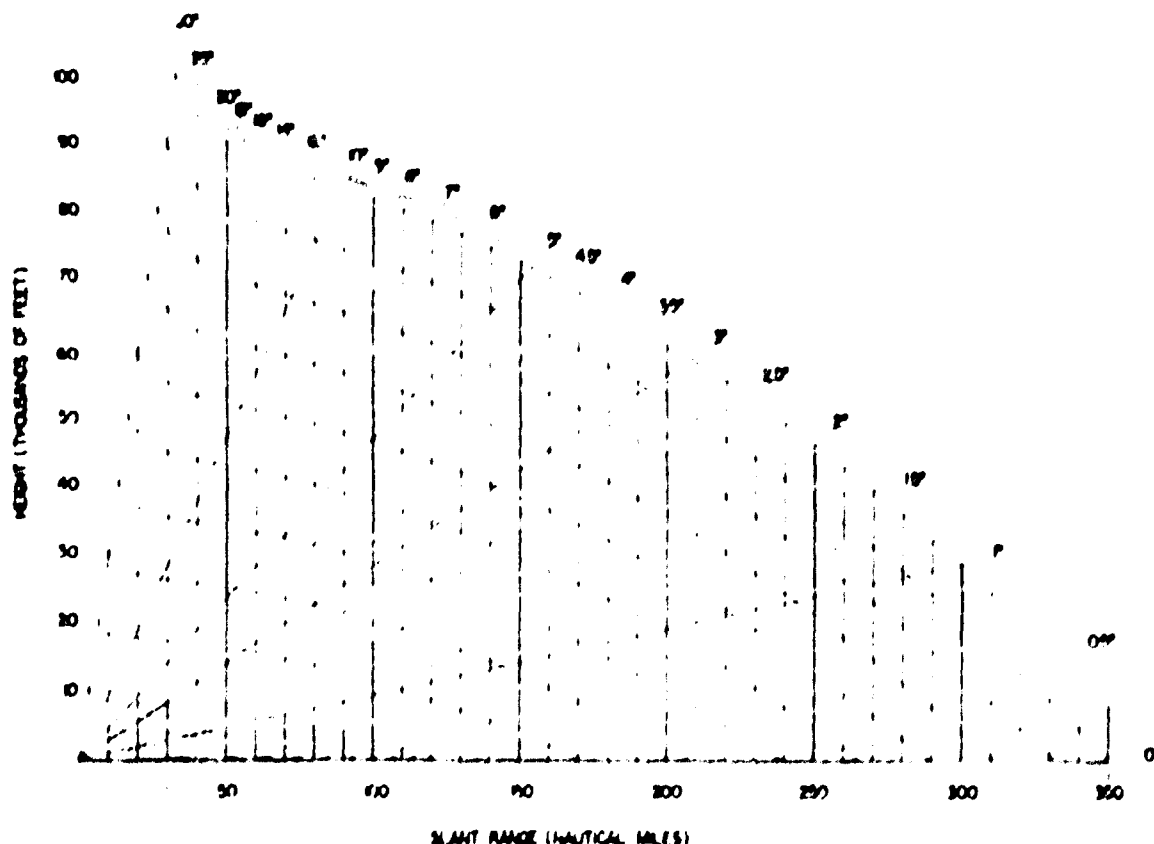
Bauer has also formulated models (1) that apply to January and July conditions in Washington, D.C. The January model is virtually the same as the April model, having almost insignificantly smaller surface refractivity. The July model is:

$$n(h) = 1 + 0.000366 \exp(0.0431 h). \quad (4)$$

It is noteworthy that the exponential constant of this July model is smaller than that of the CRPL year-round-average model, Eq. (2), illustrating the basis of Bauer's view that the CRPL exponential constant is unrealistically high. On the other hand it is also noteworthy that the CRPL model results in a surface gradient of the refractive index that is intermediate between the gradients of Bauer's April model, Eq. (1), and his July model, Eq. (4).

The statistical nature of the earth's surface and the atmosphere's behavior allows various viewpoints as to the proper ground rules for computing average behavior - i.e., whether it should be computed for sea level or for average terrain height, etc. But, this same statistical nature insures that however such questions are decided within broad limits, any reasonable exponential model will hold within the range of variation encountered.

²In a private communication dated April 14, 1961. Here only a brief record of Bauer's comments are given, in which it is hoped the essential points are covered.



At the end of this report a reproducible copy of the range-height chart for this model will be found. This copy may be reproduced without specific permission. (A similarly reproducible and reproducible chart for Bauer's model, Fig. 2, was provided in an earlier NBS report (3).) If charts are desired covering other limits of range, height, and angle, they may be constructed by the method described in Ref. 2 from Table 1, if the units desired are nautical miles, feet, and degrees. If a chart in terms of kilometers and milliradians is desired, tables of values will be found in NBS Monograph 4, Ref. 5.

The author wishes to emphasize the primary role, in the topic that has been discussed, of the published work of Bauer et al. of the MIT Lincoln Laboratory and of Bean and Thayer of the National Bureau of Standards (NBS). It is hoped that the present report may be a

step toward the fullest utilization of their important work, especially as it applies to the design and operation of radar systems. The same statement applies to earlier publications (2,3,6) in which some of this material has appeared. Special thanks are extended to B. R. Bean and J. R. Bauer for their helpful replies to queries concerning the problem of selecting a specific refractivity model.

Acknowledgment with thanks is also made of the assistance of the NRL Research Computation Center, and especially of Mrs. Dolores Milko, in the programming for the NAREC computer of the ray-path calculations, which resulted in Fig. 1 and Table 1.

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Table 1
Ray Paths Calculated for Proposed Standard Model of Atmospheric Refractive Index
(CRPL Exponential Reference Atmosphere for $N_0 = 313$)

Values of range in nautical miles, for ray of specified initial elevation angle,
at selected heights

Height (feet)	Initial Elevation Angle (degrees)								
	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
1,000	39.80	15.86	8.952	6.137	4.651	3.740	3.125	2.684	2.351
2,000	56.20	28.19	17.10	12.00	9.179	7.414	6.212	5.343	4.686
3,000	68.73	38.61	24.63	17.61	13.59	11.03	9.262	7.978	7.004
4,000	79.25	47.80	31.65	23.01	17.89	14.53	12.27	10.59	9.306
5,000	88.49	56.09	38.25	28.21	22.09	18.07	15.25	13.18	11.59
6,000	96.82	63.70	44.48	33.22	26.19	21.51	18.19	15.74	13.86
7,000	104.4	70.77	50.41	38.07	30.21	24.89	21.10	18.28	16.12
8,000	111.5	77.40	56.07	42.77	34.13	28.22	23.97	20.80	18.36
9,000	118.1	83.64	61.49	47.33	37.97	31.50	26.82	23.30	20.58
10,000	124.4	89.57	66.70	51.76	41.73	34.73	29.63	25.78	22.79
20,000	174.1	137.6	110.5	90.59	75.80	64.67	56.11	49.41	44.05
30,000	211.3	174.1	145.2	122.6	105.0	91.15	80.13	71.24	63.98
40,000	242.2	204.5	174.5	150.4	130.9	115.1	102.2	91.56	82.74
50,000	269.2	231.3	200.4	175.1	154.2	137.0	122.6	110.6	100.5
60,000	293.3	255.2	223.8	197.6	175.7	157.3	141.8	128.6	117.4
70,000	315.3	277.0	245.2	218.4	195.7	176.3	159.8	145.7	133.5
80,000	335.6	297.3	265.1	237.8	214.4	194.3	176.9	161.9	148.9
90,000	354.7	316.2	283.8	256.0	232.0	211.3	193.2	177.5	163.7
100,000	372.6	334.1	301.4	273.3	248.8	227.5	208.8	192.5	178.1
200,000	516.1	477.2	443.2	413.1	386.0	361.5	339.2	319.9	300.5
300,000	625.5	586.4	551.5	520.9	492.6	466.6	442.6	420.4	399.3
400,000	717.7	678.4	643.6	612.1	583.1	556.2	531.1	507.7	485.8
500,000	799.0	759.7	724.7	692.8	663.2	635.7	610.0	585.7	562.9
600,000	872.6	833.3	798.1	765.0	736.1	708.1	681.8	656.9	633.3
700,000	940.6	901.2	865.9	833.5	803.3	775.0	748.2	722.9	698.8
800,000	1004	964.5	929.1	896.6	866.2	837.6	810.5	784.7	760.1
900,000	1064	1024	988.8	956.1	925.5	896.6	869.2	843.1	818.1
1,000,000	1120	1081	1045	1013	981.8	952.7	925.0	898.6	873.3

(Table Continued)

Table 1 (Continued)
 Ray Paths Calculated for Proposed Standard Model of Atmospheric Refractive Index
 (CRPL Exponential Reference Atmosphere for $N_0 = 313$)

Values of range in nautical miles, for ray of specified initial elevation angle,
 at selected heights

Height (feet)	Initial Elevation Angle (degrees)								
	4.5	5.0	6.0	7.0	8.0	9.0	10	15	20
1,000	2.092	1.884	1.572	1.349	1.182	1.051	0.947	0.636	0.481
2,000	4.172	3.760	3.139	2.695	2.361	2.101	1.893	1.271	0.962
3,000	6.241	5.627	4.701	4.077	3.538	3.150	2.839	1.906	1.443
4,000	8.298	7.486	6.259	5.377	4.714	4.197	3.783	2.541	1.924
5,000	10.34	9.336	7.811	6.713	5.887	5.242	4.726	3.176	2.404
6,000	12.38	11.18	9.358	8.047	7.058	6.286	5.667	3.810	2.885
7,000	14.4	13.01	10.90	9.377	8.227	7.328	6.608	4.444	3.365
8,000	16.41	14.84	12.44	10.70	9.393	8.377	7.548	5.077	3.845
9,000	18.41	16.65	13.97	12.03	10.56	9.409	8.486	5.710	4.326
10,000	20.40	18.46	15.50	13.35	11.72	10.45	9.424	6.343	4.805
20,000	39.69	36.09	30.49	26.37	23.22	20.74	19.73	12.65	9.597
30,000	57.97	52.93	45.00	39.07	34.50	30.87	27.92	18.93	14.37
40,000	75.33	69.05	59.04	51.46	45.55	40.84	37.00	25.16	19.13
50,000	91.89	84.52	72.34	63.54	56.39	50.65	45.95	31.36	23.88
60,000	107.7	99.40	85.84	75.34	67.03	60.31	54.79	37.52	28.61
70,000	122.9	113.8	98.07	86.87	77.46	69.82	63.51	45.65	33.32
80,000	137.6	127.6	111.1	98.15	87.72	79.19	72.12	49.73	38.02
90,000	151.7	141.1	123.3	109.2	97.79	89.43	80.63	55.79	42.70
100,000	165.3	154.1	135.2	120.0	107.7	97.53	89.04	61.80	47.37
200,000	283.6	268.2	241.1	218.3	199.0	182.4	168.2	120.1	93.20
300,000	380.7	363.0	331.2	303.6	279.6	258.6	240.2	175.5	137.6
400,000	465.3	446.1	411.0	380.1	352.7	328.4	306.8	228.4	180.3
500,000	541.3	521.0	483.6	450.1	420.1	393.2	369.0	279.1	222.9
600,000	611.0	589.8	550.6	515.1	487.1	454.0	427.7	328.0	263.9
700,000	675.8	653.9	613.2	576.1	542.4	511.6	483.5	375.1	303.9
800,000	736.7	714.2	672.3	633.9	598.7	566.4	536.7	420.8	343.1
900,000	794.3	771.4	728.4	688.8	652.4	618.8	587.8	465.1	381.5
1,000,000	849.1	825.8	782.0	741.4	703.9	669.1	637.0	508.2	419.1

Table 1 (Continued)
 Ray Paths Calculated for Proposed Standard Model of Atmospheric Refractive
 Index (CRPL Exponential Reference Atmosphere for $N_0 = 313$)

Values of range in nautical miles for ray of specified initial elevation
 angle, at selected heights

Height (feet)	Initial Elevation Angle (degrees)							
	25	30	40	50	60	70	80	90
1,000	0.369	0.329	0.256	0.215	0.190	0.175	0.167	0.165
2,000	0.779	0.658	0.512	0.430	0.380	0.350	0.334	0.329
3,000	1.168	0.987	0.768	0.644	0.570	0.525	0.501	0.494
4,000	1.557	1.310	1.024	0.859	0.760	0.701	0.669	0.658
5,000	1.946	1.645	1.280	1.074	0.950	0.876	0.836	0.822
6,000	2.335	1.974	1.536	1.289	1.140	1.051	1.003	0.987
7,000	2.724	2.303	1.792	1.504	1.330	1.226	1.171	1.152
8,000	3.113	2.632	2.048	1.719	1.520	1.401	1.337	1.317
9,000	3.502	2.961	2.304	1.933	1.710	1.576	1.504	1.481
10,000	3.891	3.290	2.560	2.148	1.900	1.751	1.671	1.646
20,000	7.775	6.576	5.118	4.296	3.800	3.503	3.342	3.292
30,000	11.65	9.858	7.675	6.443	5.760	5.254	5.013	4.937
40,000	15.52	13.14	10.23	8.589	7.600	7.005	6.685	6.583
50,000	19.38	16.41	12.78	10.74	9.499	8.756	8.356	8.229
60,000	23.24	19.68	15.34	12.88	11.40	10.51	10.03	9.875
70,000	27.08	23.94	17.89	15.02	13.30	12.26	11.70	11.52
80,000	30.92	26.20	20.43	17.17	15.20	14.01	13.37	13.17
90,000	34.75	29.46	22.98	19.31	17.09	15.76	15.04	14.81
100,000	38.57	32.71	25.53	21.45	18.99	17.51	16.71	16.46
200,000	76.35	64.97	50.89	42.83	37.95	35.01	33.42	32.92
300,000	113.4	96.82	76.09	64.15	56.88	52.50	50.13	49.37
400,000	149.6	128.3	101.1	85.39	75.79	69.97	66.83	65.83
500,000	185.5	159.3	126.0	106.6	94.66	87.44	83.53	82.29
600,000	220.6	190.0	150.8	127.7	113.5	104.9	100.2	98.75
700,000	255.1	220.4	175.4	148.7	132.3	122.3	116.9	115.2
800,000	289.2	250.4	199.8	169.7	151.1	139.8	133.6	131.7
900,000	322.7	280.1	224.1	190.7	169.9	157.2	150.3	148.1
1,000,000	355.7	309.5	248.3	211.5	188.6	174.6	167.0	164.6

